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December 6, 1996

Mr. William S. Caton
Federal Communications Commission
1919 M Street, N.W.
Washington, D.C. 20554

Re: Notification of Ex Parte Contact in ET Docket No. 96-102

Dear Mr. Caton:

This letter is to notify you of a written ex parte contact in ET Docket No. 96-102. Today, copies of the attached letter were distributed to Dr. Michael Marcus from the Office of Engineering & Technology and to Thomas Tycz, Harold Ng, and Karl Kensinger from the International Bureau.

Should any questions arise concerning this letter, please contact the undersigned at (202) 828-3182.

Sincerely,

Eric W. DeSilva

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WINFORUM

Wireless Information Networks Forum, Inc.

December 6, 1996

Dr. Michael J. Marcus
Office of Engineering and Technology
Federal Communications Commission
Washington, DC 20554

Re: Amendment of the Commission's Rules to Provide for Unlicensed NII/SUPERNet Operations in the 5 GHz Frequency Range
ET Docket 96-102

Dear Dr. Marcus:

This is to further address an issue related to concerns of potential interference from NII/SUPERNet devices to the Mobile Satellite Service (MSS) feeder uplink in the 5150-5250 MHz band. In a letter to you on November 1, WINForum and Apple Computer, Inc. jointly proposed some specific limits on operating parameters for NII/SUPERNet devices intended to protect MSS operations from harmful interference while allowing NII/SUPERNet devices adequate design flexibility. The parties proposed that the limit on RF power output be specified in terms of transmit power (*i.e.*, into the antenna terminals) rather than as an EIRP limit. However, we understand that some questions remain regarding a transmit power limit versus an EIRP limit. Specifically, MSS interests have raised the concern that a transmit power limit would allow high EIRP, and that if high-gain antennas are systematically directed horizontally (as would be expected), the EIRP as seen by a satellite at a low elevation angle could cause harmful interference to the MSS feeder uplink.

To explore this concern, we have conducted detailed analyses to investigate the average NII/SUPERNet device antenna gain as seen by a low earth orbit satellite. Our analyses, described in attachments 1 and 2 to this letter, show that such a concern is unjustified. In fact, the opposite is true. The average gain seen by the satellite is actually lower for high gain antennas. In Attachment 1, three NII/SUPERNet antenna types are compared: (1) the antenna pattern introduced by AirTouch in its Reply Comments, with a range of beamwidths; (2) a parabolic dish with various diameters; and (3) a half-wave dipole. For antennas with

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significant directivity, the average gain was found to be less than 0 dBi. Of all the antennas studied, the half-wave dipole exhibited the highest average gain (about 0.8 dBi).

In Attachment 2, specific satellite positions over the continental U.S. were explored to determine the maximum interference power that could be received by the satellite, assuming high NII/SUPERNet device concentrations on both coasts. The AirTouch antenna formula was used for the NII/SUPERNet devices. The maximum average gain found was about 1 dBi for a worst-case satellite position and antenna gain. However, due to the high speed of the satellite with respect to the Earth's surface, such a worst-case satellite position would be highly transient.

We conclude that overall, with a mix of antenna types and orientations, the average antenna gain seen by a low-earth orbit satellite will be less than 0 dBi, and the interference received by the satellite will depend on the average transmit power, not the EIRP, of the NII/SUPERNet devices. Therefore, it is the transmit power that should be regulated in the Commission's Rules. In fact, with a transmit power limit, the interference to MSS is likely to be *less* than with an EIRP limit, because a transmit power requirement would provide some incentive for designers to use directive antennas, for which the average power to the satellite would be less than for a non-directive antenna with the same input power.

We would be happy to discuss any of this material with you at your convenience.

Respectfully submitted,

WIRELESS INFORMATION NETWORKS FORUM

By: ______________________
Eric W. DeSilva

cc: ET Docket 96-102
Thomas Tycz, FCC-IB
Harry Ng, FCC-IB
Karl Kensinger, FCC-IB

Average Antenna Gain for NII/SUPERNet Devices

Analysis of the effect of average NII/SUPERNet antenna gain to MSS feeder uplinks using the gain pattern of the AirTouch comments on ET Docket 96-102

Donald C. Johnson

Abstract

This paper shows that antenna gain of NII/SUPERNet devices has little effect on the mean signal level these devices will generate in the MSS feeder uplink band. In most cases this level is less with high gain antennas than with the same average power level and low gain antennas.

The signal level created by NII/SUPERNet devices at the MSS satellite is the result of a large number of device transmissions, thus if antennas with high gain are considered to be pointed in totally random directions the overall average gain is the same as if all antennas were omnidirectional. Any situation in which the average gain is higher than unity must result from some systematic pointing arrangement. This paper investigates the effect when all high gain antennas are pointed in a horizontal direction which is the only likely direction which may be favored.

The antenna gain template used in the AirTouch comments on the NII/SUPERNet docket (FCC ET Docket 96-102) is used.

The MSS satellite receiver antenna pattern is assumed to extend to a range at which the earth radio observer elevation angle to the satellite is 10 degrees. Depending on refraction conditions, this is shown to cover a radius of 1500 to 2200 miles from beneath the satellite. Thus, in some conditions, the satellite coverage includes the whole North American continent.

Device density patterns in terms of percent of devices at each vertical pointing angle are developed representing the worst case under each of 2 radio diffraction conditions and the overall average gain in the direction of the satellite is computed with each density pattern. These worst density patterns only occur at specific satellite locations. They represent the cases where most devices are at the maximum and most sensitive distance from the satellite.

At the worst satellite positions, the highest average gain of the horizontally pointing antennas is 1.0 dB (relative to isotropic) and occurs at a vertical beamwidth of 28° in the worst diffraction condition. In most density situations, a collection of antennas with gain greater than 1 systematically pointing in the horizontal direction creates an average gain less than 1 (0 dBi). Thus, limiting the antenna gain for NII/SUPERNet devices (by specifying a limit on EIRP rather than transmit power) will very likely result in higher mean signal level at the MSS satellite.

Outline

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Appendix 1. Computation of Elevation Angle vs. Distance

Appendix 2. Example Computation of Average Gain using the Case1 and Case 2 Device Densities.

Appendix 3. Results of Average Gain Computation at Better Resolution for the Numerical Integration

Appendix 4. Some Information on Parabolic Antennas

1.0 Purpose

The appendix to the AirTouch reply comments on ET docket 96-102 (hereinafter referred to as the AirTouch analysis) contains an analysis of the average antenna gain of a large number of NII/SUPERNet devices in the direction of a low orbit satellite with the NII/SUPERNet devices configured with horizontally pointing directional antennas. This analysis looks at this situation in more detail to show the effect of high gain NII/SUPERNet antennas on the mean signal level at the MSS satellite.

The deployment assumptions, transmitter duty cycle and other arrangements of the AirTouch analysis are questionable, but are not challenged here; the scope of this paper is limited to the effect of antenna gain.

The satellite receiver has an iso - flux antenna pattern that provides equal attenuation to devices with an elevation angle toward the satellite from directly beneath (90 degrees) to as low as 10 degrees. The satellite altitude is 879 miles; at this altitude a 10 degree elevation angle indicates that the iso-flux pattern extends to about 1500 to 2200 miles from directly beneath the satellite. This means that a satellite over the north central region of the US would cover the whole continent in an iso -flux manner.

This analysis assumes the same relative NII/SUPERNet device antenna gain pattern as the AirTouch analysis and extends the analysis by evaluating some actual device density situations and providing more detail.

The conclusion is that the worst average gain is about 1 dB, relative to an isotropic antenna, and that in the typical situations, the average gain is less than 0 dB with high gain antennas.

2.0 The Antenna Pointing Arrangement

At the power levels proposed for NII/SUPERNet devices in the band shared with MSS it would require a very large number of devices transmitting simultaneously to cause a measurable level at the satellite. Thus, if all device antennas are arranged in totally random directions the average gain is unity regardless of the individual antenna gains. Only if the device antennas are systematically pointed in a direction toward the satellite will the average gain be greater than unity.

The only likely systematic pointing direction is horizontal at the location of the device. Since the vertical angle to the satellite is as low as 10 degrees at the most distant locations, devices with gain greater than unity pointing horizontally may generate average gains greater than unity in the direction of the satellite in some cases. This paper investigates those cases.

If the devices are used in an outside point-to-point link, their pointing direction will likely be horizontal. In this case, the gain of the antennas will likely be as high as practical and permitted by the rules. If the devices are used inside buildings (the principal intended use), access points or base stations with gain above 0 dBi may tend to point horizontally more than in other directions. However, in most cases the mobile pointing directions will be fully random. Thus, for inside devices the assumption that all antennas point in the horizontal direction represents an extreme worst case.

In sum, the pointing arrangement posited may apply to high gain outside point-to-point devices or to some portion of low and intermediate gain inside devices.

3.0 The Average Gain From the AirTouch Analysis.

The AirTouch analysis give an analytical expression for device antenna gain. This is the pattern template used here.

This pattern is a good representation for the purposes of defining average gain of distributed devices for the relative low gain that will normally be encountered in portable devices. However, the pattern does not cut off as sharply for higher gain antennas in the 5.2 GHz range as do most practical real antennas and is likely to overestimate the sidelobe power at high gain.

The analytical expression proposed represents a gain which averages more than 1 over a sphere and must be corrected by a factor in order to represent a real antenna. It is shown that with this correction, the average gain of the AirTouch analysis is always less than 1 dB.

The gain equation of the AirTouch analysis permits defining a horizontal and vertical beamwidth, but for purposes of calculating average gain it is observed that the average gain in any vertical direction with the template pattern is almost independent of the horizontal beamwidth (see following note). Thus, for the principal calculations here, the horizontal and vertical beamwidths are set equal.

From the AirTouch analysis, page 3 with $B_{w\varepsilon} = B_{w\alpha}$.

$$G_0 = \frac{27{,}000}{Bw_\varepsilon Bw_\alpha} \text{ and with the above assumption} \qquad (1)$$

$$Bw_\varepsilon = Bw_\alpha = Bw \qquad (2)$$

$$G_0 = \frac{27{,}000}{Bw^2}\text{. Define} \qquad (3)$$

$$M = 10^{\frac{-1}{2Bw^2}}\text{, then} \qquad (4)$$

$$G_a(\varepsilon,\alpha) = G_0 M^{\varepsilon^2} M^{\alpha^2} + 1 \qquad (5)$$

Equation 5 is the special case of the AirTouch gain expression when the horizontal and vertical beamwidths are equal. The general equation is:

$$G_a(\varepsilon,\alpha) = G_0 10^{-1/2(\varepsilon/B_{w\varepsilon})^2} 10^{-1/2(\alpha/B_{w\alpha})^2} + 1 \qquad (5a)$$

Note: That the average gain in any vertical direction is almost independent of horizontal beamwidth can be shown by inspecting equations 1 and 5a. From 5a, the horizontal beamwidth at any vertical angle is equal to $B_{w\alpha}$. Thus, the average of this term over all values of α is directly proportional to $B_{w\alpha}$. From 1, G_0 is inversely proportional to $B_{w\alpha}$. Thus, the product is independent of B_{wa} as is the complete expression 5a.

$G_a(\varepsilon, \alpha)$ is the gain of each individual antenna in the AirTouch analysis. However, the average of this gain is greater than unity, thus a correction is needed.

The AirTouch analysis evaluates an average gain toward the satellite for a collection of antennas pointing in evenly distributed random horizontal directions in accordance with the following expression.

$$G_{avg}(\varepsilon_1) = \frac{1}{2\pi(1 - Sin\varepsilon_1)} \int_{-\pi}^{\pi} \int_{\varepsilon_1}^{\pi/2} G(\varepsilon,\alpha) Cos\varepsilon \; d\varepsilon d\alpha \,, \qquad (6)$$

where ε_1 is the elevation angle from the most distant devices to the satellite and $G(\varepsilon, \alpha)$ is the gain of each antenna. The AirTouch equation uses G_a as the antenna gain. This is not used here because G_a has an average in excess of 1.

This is a correct expression for the gain of a collection of antennas at a given point in the sense that it is the ratio of the total power directed at an angle of ε_1 and above to the power that would be radiated in that direction by an ideal omni antenna. The AirTouch conclusion is that the maximum of this mean stated in deciBels is 2. However, some reflection will show that the value cannot be greater than 1 (0 dB) if the gain G is a real antenna gain in the sense that it averages 1 over the surface of a sphere. By the basic gain definition, a higher density of power flows in the direction $-\varepsilon_1$ to $+\varepsilon_1$ than flows at angles above or below ε_1. thus, the average gain above ε_1 must be less than 1.

Note: The expression is the average gain, of the random collection of antennas, in the direction of the partial spherical surface that consists of the surface above the angle ε_1 if the collection of antennas are considered to be located at a point. Thus, if the surface of this partial sphere contained an even density of emitters of gain G, all pointing horizontally, and the point collection of antennas are treated as a single omni directional receiver, the mean gain of the collection of emitters toward the receiver would be as given. This structure can then be inverted and the satellite can be considered the receiver. The iso-flux nature of the actual satellite receiver makes all devices appear to be at equal distance, thus the earth surface appears as the lower portion of a partial sphere relative to distance attenuation.

The expression would be correct if there were always the same number of devices within a solid angle of width $\delta\varepsilon\delta\alpha$ about the satellite and within the iso-flux pattern. This cannot be expected to be the case, however. For example, a satellite near the east coast might sense about all of the west coast devices at an elevation near $\varepsilon_1 = \pi/18$ (10 degrees) plus some devices in Canada, Mexico and South America (10 degrees is about 1500 to 2000 miles). However, the number of devices on the east coast within a circle (of much lower diameter) would only contain two small areas that include the dense east coast populated region. Thus, this satellite would experience a high density at low elevation angle and a low density at larger angles (shorter distance).

Further, if the density over the earth surface is constant, the number of devices in an angle of width $\Delta\varepsilon$ increases with decreasing angle. Thus, the number of devices in the angular width $\Delta\varepsilon$ is larger at longer distances.

G_{avg} is not the actual average gain, but a correction factor can be applied to it to make the average unity.

Define,

$$I_s(\varepsilon_1) = \int_{-\pi}^{\pi} \int_{\varepsilon_1}^{\pi/2} G_a(\varepsilon,\alpha) Cos\varepsilon \; d\varepsilon d\alpha \,. \qquad (7)$$

If the actual antenna gain = $k_1 G_a$ then the actual average antenna gain over the complete spherical surface is:

$$G_{avg}(-\pi/2) = \frac{k_1}{4\pi} I_s(-\pi/2) = 1$$

Then

$$k_1 = 4\pi / I_s(-\pi/2) \,, \qquad (8)$$

and with $G_{avg}(\varepsilon_1)$ = the actual average gain under the assumptions of the above note, then.

$$G_{avg}(\varepsilon_1) = \frac{k_1}{2\pi(1 - Sin\varepsilon_1)} I_s(\varepsilon_1) \qquad (9)$$

This average gain expression is only accurate for a particular device density distribution over the earth surface. It is the average gain of the antenna collection at angles above ε_1. If there are more devices at some angles than at others, this will affect the actual average gain.

Evaluation of I_s.

Equation 7 can be evaluated using numerical integration. This will be done next. The results can also be used to evaluate the average gain with other device density distributions to be investigated in a subsequent section.

Define F(α,ε) as follows:

$$I_s(\varepsilon_1) = \int_{-\pi}^{\pi} F(\alpha, \varepsilon_1) d\alpha$$

$$F(\alpha, \varepsilon_1) = \int_{\varepsilon_1}^{\pi/2} G_a(\varepsilon, \alpha) Cos\varepsilon \, d\varepsilon .$$

This can be further evaluated to

$$F(\alpha, \varepsilon_1) = G_0 M^{\alpha^2} \int_{\varepsilon_1}^{\pi/2} M^{\varepsilon^2} Cos\varepsilon \, d\varepsilon + 1 - Sin\varepsilon_1$$

$$I_1(\varepsilon_1) = \int_{\varepsilon_1}^{\pi/2} M^{\varepsilon^2} Cos\varepsilon \, d\varepsilon \qquad (10)$$

$$F(\alpha, \varepsilon_1) = G_0 M^{\alpha^2} I_1(\varepsilon_1) + 1 - Sin\varepsilon_1$$

$I_s(\varepsilon_1)$ can be further reduced to

$$I_s(\varepsilon_1) = G_0 I_1(\varepsilon_1) \int_{-\pi}^{\pi} M^{\alpha^2} d\alpha + 2\pi(1 - Sin\varepsilon_1).$$

Let

$$I_2 = \int_{-\pi}^{\pi} M^{\alpha^2} d\alpha \text{ , then} \qquad (11)$$

$$I_s(\varepsilon_1) = G_0 I_1(\varepsilon_1) I_2 + 2\pi(1 - Sin\varepsilon_1).$$

Then $I_1(\varepsilon_1)$ and I_2 can be evaluated by numeric integration.

The average gain at angles above ε_1 is given by the following:

$$G_{avg}(\varepsilon_1) = k_1 \left[1 + \frac{G_o I_1(\varepsilon_1) I_2}{2\pi(1 - Sin\varepsilon_1)} \right] \qquad (12)$$

4.0 The Average Gain Versus Elevation Angle

To understand the effect of actual potential device distributions it is instructive to investigate the average gain at specific elevation angles and then consider the actual anticipated device density. Call this average gain over all horizontal directions $G_e(\varepsilon)$.

Consider as before, a collection devices with antennas pointing in evenly distributed horizontal directions with gain $G(\alpha,\varepsilon)$, where α is the horizontal angle and ε is the vertical angle. The power flowing out of a small vertical angle $\Delta\varepsilon$ is

$$P_e(\varepsilon)=\frac{P_g}{4\pi r^2}\int_{\varepsilon_1-\Delta\varepsilon/2}^{\varepsilon_1+\Delta\varepsilon/2}\int_{-\pi}^{\pi} r^2 G(\alpha,\varepsilon)Cos\varepsilon d\alpha d\varepsilon=\frac{P_g}{4\pi}\int_{\varepsilon_1-\Delta\varepsilon/2}^{\varepsilon_1+\Delta\varepsilon/2}\int_{-\pi}^{\pi} G(\alpha,\varepsilon)Cos\varepsilon d\alpha d\varepsilon\ ,$$

where P_g is the power generated by the collection devices and r is the radial distance.

The integration over α will yield some function $F(\varepsilon)$. Then, the integration of $F(\varepsilon)$ over the differential limit range will yield $F(\varepsilon)d\varepsilon$. Thus, $P_e(\varepsilon) = (P_g/4\pi)F(\varepsilon)d\varepsilon$. The ratio of the power through the differential angle with gain G to that with a gain of 1 is the power gain at the angle ε.

This power per unit elevation angle with a gain of 1 is easily shown to be $(P_g/2)Cos\varepsilon$.

Now consider a gain = $k_1G_a(\alpha,\varepsilon)$.

From (5)

$$G_a(\varepsilon,\alpha)=G_0M^{\varepsilon^2}M^{\alpha^2}+1$$

Then $F(\varepsilon)$ becomes

$$F(\varepsilon)=k_1Cos\varepsilon\left[G_oM^{\varepsilon^2}\int_{-\pi}^{\pi}M^{\alpha^2}d\alpha+\int_{-\pi}^{\pi}d\alpha\right].$$

Then using (11)

$F(\varepsilon)=k_1Cos\varepsilon\left[G_oM^{\varepsilon^2}I_2+2\pi\right]$, when the gain is $k_1G_a(\alpha,\varepsilon)$.

The power density per unit elevation angle is $(P_g/4\pi)F(\varepsilon)$. This quantity divided by the power density with unity gain is the average gain over all horizontal directions at the angle ε. This was named $G_e(\varepsilon)$ above. This ratio is

$$G_e(\varepsilon)=k_1\left[1+G_oM^{\varepsilon^2}\left(\frac{I_2}{2\pi}\right)\right] \qquad (13)$$

5.0 Some Example Computations of Gain Versus Elevation Angle

The following equations were used in evaluating the integrals. The accuracy was checked with a Basic program with better resolution, however.

$$I_1(\varepsilon_1)\approx\Delta\varepsilon\sum_{n=\varepsilon_1/\Delta\varepsilon}^{n=(\pi/2\Delta\varepsilon)-1}M^{\left[\frac{180(n+.5)\Delta\varepsilon}{\pi}\right]^2}Cos[(n+.5)\Delta\varepsilon]$$

$$I_2\approx2\Delta\alpha\sum_{n=0}^{n=(\pi/\Delta\alpha)-1}M^{\left[\frac{180(n+.5)\Delta\alpha}{\pi}\right]^2}$$

The values of $\Delta\varepsilon$ and $\Delta\alpha$ were 0.0315 radians, corresponding to 1.803 degrees. This provides sufficient resolution to accurately reflect the gains at solid angles down to the 5 degree values shown in the following tables.

Note: The gain values were checked with a program written in MS Quick Basic and the accuracy was verified. This program was also used to verify the independence of the average gain with horizontal beamwidth.

Tables 1 through 4 show some example computations. Note that G_{avg} is less than 1 in all cases shown. However, the average gain at specific values of ε exceeds 1 at low elevation angles.

Table 1

B_w = 60 degrees $I_2(B_w) = 1.691$ $G_o(B_w) = 7.500$ $k_1(B_w) = 0.419$ Maximum gain = 5.52 dB

ε_1 degrees	$I_1(\varepsilon_1)$	$I_s(\varepsilon_1)$	G_{avg} (ratio)	G_{avg} (dB)	$G_e(\varepsilon_1)$ (ratio)	$G_e(\varepsilon_1)$ (dB)
5	0.6078	13.62	0.9826	-0.08	1.26	1.00
10	0.5158	11.88	0.9471	-0.24	1.24	0.93
15	0.4569	10.58	0.9404	-0.27	1.21	0.82
20	0.3737	8.98	0.8990	-0.46	1.16	0.66
25	0.2982	7.50	0.8550	-0.68	1.11	0.46
30	0.232	6.15	0.8097	-0.92	1.05	0.23
35	0.1927	5.18	0.7998	-0.97	0.99	-0.04
40	0.1423	4.09	0.7541	-1.23	0.93	-0.33
45	0.1014	3.16	0.7097	-1.49	0.86	-0.64

Table 2

B_w = 30 degrees $I_2(B_w) = 0.865$ $G_o(B_w) = 30.0$ $k_1(B_w) = 0.373$ Maximum gain = 10.6 dB

ε_1 degrees	$I_1(\varepsilon_1)$	$I_s(\varepsilon_1)$	G_{avg} (ratio)	G_{avg} (dB)	$G_e(\varepsilon_1)$ (ratio)	$G_e(\varepsilon_1)$ (dB)
5	0.345	14.69	0.954	-0.20	1.864	2.70
10	0.256	11.84	0.850	-0.71	1.727	2.37
15	0.203	9.92	0.794	-1.00	1.527	1.84
20	0.135	7.65	0.690	-1.61	1.296	1.12
25	0.085	5.82	0.598	-2.23	1.065	0.27
30	0.049	4.41	0.524	-2.81	0.860	-0.66
35	0.033	3.53	0.491	-3.09	0.694	-1.59
40	0.017	2.68	0.445	-3.52	0.572	-2.43
45	0.008	2.04	0.414	-3.83	0.488	-3.11

Table 3

B_w = 15 degrees $I_2(B_w) = 0.433$ $G_o(B_w) = 120$ $k_1(B_w) = 0.0.362$ Maximum gain = 16.4 dB

ε_1 degrees	$I_1(\varepsilon_1)$	$I_s(\varepsilon_1)$	G_{avg} (ratio)	G_{avg} (dB)	$G_e(\varepsilon_1)$ (ratio)	$G_e(\varepsilon_1)$ (dB)
5	0.151	13.59	0.859	-0.66	3.00	4.77
10	0.075	9.11	0.636	-1.97	2.16	3.34
15	0.041	6.81	0.530	-2.76	1.31	1.17
20	0.014	4.84	0.424	-3.72	0.75	-1.26
25	0.003	3.80	0.380	-4.20	0.48	-3.15
30	0.00064	3.17	0.366	-4.36	0.39	-4.06
35	0.00018	2.69	0.364	-4.39	0.37	-4.34
40	0.00002	2.25	0.363	-4.41	0.36	-4.40
45	0.00000	1.84	0.362	-4.41	0.36	-4.41

Table 4

B_w = 7.5 degrees $I_2(B_w) = 0.216$ $G_o(B_w) = 480$ $k_1(B_w) = 0.360$ Maximum gain = 22.4 dB

ε_1 degrees	$I_1(\varepsilon_1)$	$I_s(\varepsilon_1)$	G_{avg} (ratio)	G_{avg} (dB)	$G_e(\varepsilon_1)$ (ratio)	$G_e(\varepsilon_1)$ (dB)
5	0.0497	10.90	0.684	-1.65	3.92	5.94
10	0.0071	5.93	0.411	-3.86	1.13	0.52
15	0.001074	4.77	0.368	-4.34	0.42	-3.78
20	0.000025	4.14	0.360	-4.44	0.36	-4.42
25	1.84E-07	3.63	0.360	-4.44	0.36	-4.44
30	4.17E-10	3.14	0.360	-4.44	0.36	-4.44

6. 0 Overall Average Gain with Typical Device Density Distributions

G_{avg} as given by equation 6 is always less than 1 (0 dBi) when the antenna gain is corrected to average 1 over a complete sphere. This is verified in tables 1 through 4 above. However, equation 6 (used in the AirTouch analysis) represents the actual average gain only under a specific assumption of device density distribution.

Here two device distributions are considered in which there are the maximum number of devices at the longer ranges that will occur over the continental US, thus creating the worst condition for average gain for the particular maximum iso-flux range considered. It is assumed that the density of devices will form the same pattern as the population density of the continental US.

In case 1, the radial distance from beneath the satellite to the point where the local elevation angle to the satellite is 10 degrees is 2200 miles. Appendix 1 shows that this will be the approximate distance with relatively high atmospheric refraction.

In case 2, this distance is considered to be about 1500 miles, which corresponds to very low atmospheric diffraction.

Case 1:

The radial distance covered is 2200 miles. At this distance, the full east coast of the US has approximately a 10 degree elevation angle to a satellite over southeastern British Columbia, Canada, which is the worst case satellite location. It is estimated from an atlas that approximately 27% of the population of the US lives within about 200 miles of the east coast, and 200 miles covers an elevation angle between 10 degrees and 15 degrees with reference to the satellite position. All of Florida is outside the 10 degree angle, but the population of Florida is included in the estimate.

The population density per unit area (and corresponding device density) at distances corresponding to an elevation angle greater than 15 degrees was considered constant. The dense west coast population is at an angle greater than about 25 degrees, but the area covered by an arc through Los Angeles contains much of the low population density of the great plains.

The assumption is that the east coast population density increases linearly from the mean of the rest of the country at 15 degrees to a maximum at 10 degrees near the coast

This seems to be the worst case location for a satellite with a 2200 mile range.

Case 2:

For a satellite range of about 1500 miles radial distance to 10 degrees, it is possible to position a satellite so that both densely populated coasts appear at about the worst case elevation angle of 10 degrees. The satellite position for this is over eastern North Dakota. In this case, approximately 41% of the US population lives within the distance range corresponding to 10 degrees to 15 degrees. The density profile used includes the Florida population, although Florida is beyond the 10 degree angle range. In case 2 also, the density is considered to be distributed evenly at ranges corresponding to elevation angles above 15 degrees.

This can be considered the worst case device density distribution.

A satellite at the position assumed will move away from the most sensitive location very quickly. The satellite speed, relative to the earth surface is about 220 miles/minute. Thus, if it is moving in a southerly direction it will move to a position in which the elevation angles to both densely populated coasts become greater than 15 degrees within about 1 minute.

In most cases the device density when the satellite coverage includes the whole continental US will be more favorable than either of the above cases. That is, there will be a lower density at the longer range of the iso-flux pattern.

Appendix 2 shows the device density distribution for both cases and includes a table showing the computation of the average gain in each case with a vertical antenna beamwidth of 28 degrees. This was determined to be the worst case beamwidth for the case 2 device density distribution, as shown in table 5 below.

Table 5. Summary of Average Antenna Gain with Case 1 and Case 2 Device Density

Beamwidth (degrees)	Maximum gain with $B_{we} = B_{wa}$ (dB)	Case 1 Average gain (dB)	Case 2 Average gain (dB)
5	25.9	-4.32	-4.30
10	19.9	-1.73	-1.16
15	16.4	-.39	0.36
20	14.0	0.12	0.86
25	12.1	0.38	1.01
27	11.5	0.44	1.03
28	11.2	0.47	**1.04**
29	10.9	0.50	1.04
30	10.6	0.52	1.03
35	**9.4**	**0.58**	0.99
40	**8.38**	**0.58**	0.91
45	7.52	0.56	0.82
50	6.79	0.51	0.72
55	6.17	0.47	0.64
60	5.64	0.42	0.56
65	5.18	0.38	0.49
70	4.79	0.33	0.42

In both cases, if the antenna horizontal and vertical beamwidths are equal, the average gain is negative for antenna gains exceeding 20 dBi. In point-to-point applications the gain will almost always exceed 20 dBi. Thus, limiting the antenna gain will actually increase the mean signal level in this application.

The maximum average gain is about 1.0 dB and this occurs at a vertical beamwidth of 28 degrees and an antenna gain of about 11 dB. This occurs at the worst case device distribution and a satellite will see its effect only in extraordinary circumstances (negative refraction conditions) and for only about 1 minute on a specific orbit. Case 1 is a more typical worst distribution for normal diffraction conditions and the maximum average gain is about 0.6 dB in this case.

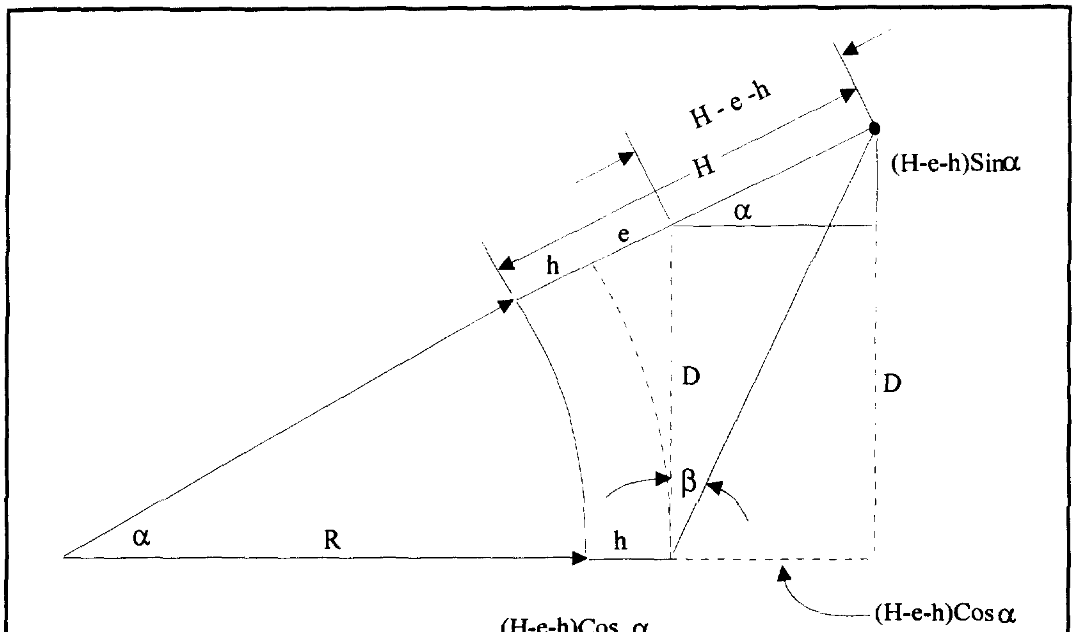
7.0 Conclusions

1. It is shown that the maximum average NII/SUPERNet device antenna gain for horizontally pointing antennas is 1 dB with the gain template used. This template can be considered to represent the actual possible antenna patterns for relatively low gain cases.

2. The worst case average gain for antennas with gains in excess of about 20 dBi is negative. This would mean that limiting the antenna gain will increase the satellite mean signal level level for point-to-point applications.

3. The average gain for fully random pointing antennas is 1 regardless of the individual antenna gains. The systematic horizontally pointing arrangement is not likely for indoor applications, so even the 1 dB average gain will not be realized in this case. The pointing arrangement over represents the actual case for overall situations.

Appendix 1. Computation of Elevation Angle vs. Distance



$$\tan\beta = \frac{(H-e-h)\cos\alpha}{D + (H-e-h)\sin\alpha}$$

Figure Ax. Antenna Pointing Angle
R = Earth Radius Times Diffraction Factor
D = Horizontal Distance to Device
h = Height of Device

This illustrates the geometry for computing the elevation angle between a device and the satellite. The device reference elevation angle pointing at the satellite is β. The angle from the satellite to the device relative to vertical at the satellite location is 90° - α - β.

The value of the earth radius is set at 4/3 (the diffraction factor) times actual to account for normal diffraction bending of the beam. R including the 4/3 factor is 5280 miles.

The value of e is:

$$e = (R + h)(Sec\alpha - 1)$$

The height of the device (h) is effectively zero relative to the satellite height and is set to zero in the computations that follow.

Appendix 1 Continued. Computations for 4/3 Earth Radius Multiplication Factor.

F = Earth radius multiplying factor for diffraction bending = 1.333

MSS Angle versus area covered — All distances in miles

H = Height of satellite = 878.6 miles

R = Earth radius 3960 miles

C = Earth distance to observer (over curved earth)

D = Horizontal distance to observer — F= **1.333**

e = Vertical distance observer to horizontal line from under satellite — e = (R+H)(SecAlpha-1)

Alpha = earth angle, observer to satellite — Alpha = Atan D/(R*F)

Angle at earth center between radial lines to two surface points

Beta = observer elevation angle to satellite

A = Area covered by radius D

Beta approximately = Atan[(H-e)CosAlpha/{D+(H-e)SinAlpha}]

A = (A/Pi)^(1/2)

A (sq miles)	C (miles)	Alpha (degrees)	e (miles)	D (miles)	Beta (degrees)	Sat angle from vertical
4.91E+06	1250.0	13.6	176.7	1280	25.29	51.15
5.11E+06	1275.0	13.8	184.0	1307	24.61	51.56
5.31E+06	1300.0	14.1	191.5	1334	23.94	51.95
5.52E+06	1325.0	14.4	199.1	1361	23.29	52.34
5.73E+06	1350.0	14.6	206.9	1388	22.65	52.70
5.94E+06	1375.0	14.9	214.9	1415	22.02	53.06
6.16E+06	1400.0	15.2	223.0	1442	21.41	53.40
6.38E+06	1425.0	15.5	231.3	1469	20.80	53.73
6.61E+06	1450.0	15.7	239.8	1497	20.21	54.05
6.83E+06	1475.0	16.0	248.4	1524	19.63	54.36
7.07E+06	1500.0	16.3	257.2	1552	19.06	54.66
7.31E+06	1525.0	16.5	266.1	1580	18.51	54.95
7.55E+06	1550.0	16.8	275.2	1607	17.96	55.22
7.79E+06	1575.0	17.1	284.5	1635	17.42	55.49
8.04E+06	1600.0	17.4	294.0	1663	16.89	55.75
8.30E+06	1625.0	17.6	303.6	1691	16.37	56.00
8.55E+06	1650.0	17.9	313.5	1720	15.86	56.24
8.81E+06	1675.0	18.2	323.4	1748	15.35	56.47
9.08E+06	1700.0	18.4	333.6	1776	14.86	56.69
9.35E+06	1725.0	18.7	344.0	1805	14.37	56.91
9.62E+06	1750.0	19.0	354.5	1833	13.89	57.12
9.90E+06	1775.0	19.3	365.2	1862	13.42	57.32
1.02E+07	1800.0	19.5	376.1	1891	12.95	57.51
1.05E+07	1825.0	19.8	387.1	1920	12.50	57.70
1.08E+07	1850.0	20.1	398.4	1949	12.04	57.88
1.10E+07	1875.0	20.3	409.8	1978	11.60	58.05
1.13E+07	1900.0	20.6	421.5	2008	11.16	58.22
1.16E+07	1925.0	20.9	433.3	2037	10.73	58.38
1.19E+07	**1950.0**	21.2	445.3	2067	**10.30**	58.54
1.23E+07	1975.0	21.4	457.5	2096	9.88	58.69

Appendix 1 Continued. Computations for Earth Radius Multiplication Factor of 1 (No Diffraction).

F = Earth radius multiplying factor for diffraction bending = 1.000

MSS Angle versus area covered — All distances in miles

H = Height of satellite = 878.62 miles

R = Earth radius 3960 miles

F = Earth radius multiplying fact for diffraction bending = **1.000**

C = Earth distance to observer (over curved earth)

D = Horizontal distance to observer

e = Vertical distance observer to horizontal line from under satellite — e = (R+H)(SecAlpha-1)

Alpha = earth angle, observer to satellite — Alpha = Atan D/(R*F)

Angle at earth center between radial lines to two surface points

Beta = observer elevation angle to satellite

A = Area covered by radius D

Beta approximately = Atan[(H-e)CosAlpha/{D+(H-e)SinAlpha}]

A = (A/Pi)^(1/2)

A (sq miles)	C (miles)	Alpha (degrees)	e (miles)	D (miles)	Beta (degrees)	Sat angle from vertical
3.14E+06	1000.0	14.5	158.5	1029	29.98	45.56
3.30E+06	1025.0	14.8	166.7	1056	29.06	46.11
3.46E+06	1050.0	15.2	175.2	1084	28.16	46.65
3.63E+06	1075.0	15.6	183.9	1111	27.29	47.16
3.80E+06	1100.0	15.9	192.9	1139	26.43	47.66
3.98E+06	1125.0	16.3	202.0	1167	25.59	48.13
4.15E+06	1150.0	16.6	211.5	1194	24.77	48.59
4.34E+06	1175.0	17.0	221.1	1222	23.96	49.04
4.52E+06	1200.0	17.4	231.0	1251	23.18	49.46
4.71E+06	1225.0	17.7	241.1	1279	22.40	49.87
4.91E+06	1250.0	18.1	251.5	1307	21.65	50.27
5.11E+06	1275.0	18.4	262.1	1336	20.91	50.65
5.31E+06	1300.0	18.8	273.0	1365	20.18	51.01
5.52E+06	1325.0	19.2	284.1	1394	19.46	51.36
5.73E+06	1350.0	19.5	295.5	1423	18.76	51.70
5.94E+06	1375.0	19.9	307.1	1452	18.08	52.03
6.16E+06	1400.0	20.3	319.0	1481	17.40	52.34
6.38E+06	1425.0	20.6	331.1	1511	16.74	52.64
6.61E+06	1450.0	21.0	343.5	1541	16.09	52.93
6.83E+06	1475.0	21.3	356.2	1571	15.45	53.21
7.07E+06	1500.0	21.7	369.2	1601	14.82	53.48
7.31E+06	1525.0	22.1	382.4	1631	14.20	53.74
7.55E+06	1550.0	22.4	395.9	1662	13.59	53.98
7.79E+06	1575.0	22.8	409.7	1692	12.99	54.22
8.04E+06	1600.0	23.1	423.7	1723	12.40	54.45
8.30E+06	1625.0	23.5	438.1	1755	11.82	54.67
8.55E+06	1650.0	23.9	452.7	1786	11.25	54.88
8.81E+06	1675.0	24.2	467.6	1817	10.69	55.08
9.08E+06	**1700.0**	24.6	482.9	1849	**10.13**	55.27
9.35E+06	1725.0	25.0	498.4	1881	9.58	55.46

With normal diffraction (F = 1.33), the distance at which the observer elevation angle (β) is 10 degrees is about 1950 miles. With no diffraction (F = 1.00) this range is 1750 miles. At either of these distances, the satellite will not be in range of the southern part of the US when the densely populated areas of the coasts are at the sensitive low angle.

The worst case device density (case 2) was based on a range at 10 degrees of 1500 miles. In this case, most of both coasts are at the 10 degree angle when the satellite is over eastern North Dakota. This range is only achieved in an abnormally high diffraction condition.

The case 1 range to 10 degrees was 2200 miles. At this range the complete east coast, including most of Florida, is approximately at the low elevation angle of 10 degrees and the satellite is over British Columbia, Canada. Negative diffraction is necessary to achieve this condition. Distances between those used will show less device density at the low end of the range of angle than will case 2.

Appendix 2. Example Computation of Average Gain using the Case1 and Case 2 Device Densities.

Case 1 device density: Low diffraction.

Diffraction multiplying factor	1.7
Distance to point of 10° elevation	2200 miles
Satellite location (worst case)	SE British Columbia

East coast of US at 10° elevation angle. 27% of devices within 200 miles of the east coast.

Case 2 device density: Normal diffraction.

Diffraction multiplying factor =	0.9
Distance to point of 10° elevation =	1500 miles
Satellite location	East Central N. Dakota

Both US coasts at 10° elevation angle. 41% of devices within 200 miles of the US coasts.

Antenna beamwidth (Bw) = 28°

		Case 1, 2200 mi radius Over BC, Canada		Case 2, 1500 mi radius Over East ND	
Angle	Ge	Density	Product		Product
10.00	1.788	7.94%	0.1419	10.58%	0.1891
11	1.745	7.07%	0.1233	8.94%	0.1560
12	1.699	6.04%	0.1026	7.43%	0.1263
13	1.652	4.87%	0.0805	6.04%	0.0997
14	1.602	3.60%	0.0576	4.76%	0.0762
15	1.550	2.23%	0.0345	3.59%	0.0557
16	1.498	2.15%	0.0322	3.41%	0.0511
17	1.445	2.08%	0.0300	3.23%	0.0467
18	1.391	2.00%	0.0279	3.06%	0.0425
19	1.337	1.93%	0.0258	2.89%	0.0386
20	1.283	1.86%	0.0238	2.73%	0.0350
21	1.230	1.92%	0.0236	2.58%	0.0318
22	1.177	1.97%	0.0232	2.44%	0.0287
23	1.126	2.01%	0.0226	2.30%	0.0259
24	1.075	2.04%	0.0219	2.17%	0.0234
25	1.027	2.06%	0.0212	2.04%	0.0210
26	0.979	1.97%	0.0193	1.93%	0.0189
27	0.934	1.89%	0.0176	1.81%	0.0169
28	0.890	1.80%	0.0160	1.71%	0.0152
29	0.848	1.72%	0.0146	1.60%	0.0136
30	0.809	1.63%	0.0132	1.50%	0.0121
31	0.771	1.63%	0.0126	1.40%	0.0108
32	0.736	1.63%	0.0120	1.31%	0.0097
33	0.703	1.63%	0.0115	1.23%	0.0086
34	0.672	1.63%	0.0109	1.14%	0.0077
35	0.643	1.59%	0.0102	1.06%	0.0068
36	0.616	1.44%	0.0089	0.99%	0.0061
>=37	0.591	29.67%	0.1753	16.12%	0.0953
Sum		100.00%	1.11		1.27
dB			0.47		1.04

Note that under typical diffraction conditions (about 2000 miles range to 10 degrees, appendix 1) when the satellite coverage covers the whole US continent, the minimum vertical angle will exceed the 10 degree assumed above. A good estimate is that if this minimum angle is greater than about 20 degrees, the overall average gain in the above case is less than 0 dB.

Appendix 3. Results of Average Gain Computation at Better Resolution for the Numerical Integration

Numerical integration at 0.1 degree increments.

The minimum vertical angle (Eps1 was)	10	degrees
The beamwidth vertical/horizontal ratio is	1	

Ge versus Epsilon at 60 and 10 degree beamwidths

Eps (degrees)	Ge (dB) at 60 degree Bw	Ge (dB) at 10 degree Bw
10	.927069	2.482751
11	.9077967	1.667826
12	.8867391	.829393
13	.8639103	-.0096
14	.8393259	-.8226725
15	.8130018	-1.581379
16	.7849562	-2.259451
17	.7552071	-2.837069
18	.7237766	-3.304688
19	.6906847	-3.664286
20	.6559554	-3.927459
21	.6196133	-4.111423
22	.5816841	-4.234791
23	.5421944	-4.314505
24	.5011736	-4.364317
25	.4586517	-4.394509
26	.4146606	-4.412298
27	.3692327	-4.422501
28	.3224028	-4.428205
29	.274207	-4.431314
30	.2246829	-4.432967
31	.1738694	-4.433825
32	.1218075	-4.43426
33	.068	-4.434475
34	.0141	-4.434579
35	-.0414	-4.434628
36	-.0980	-4.43465
37	-.1557086	-4.43466
38	-.2143265	-4.434665
39	-.2738672	-4.434667
40	-.3342778	-4.434668
41	-.3955051	-4.434668
42	-.4574935	-4.434668
43	-.5201863	-4.434669
44	-.583526	-4.434669
45	-.6474535	-4.434669
46	-.711909	-4.434669
47	-.7768312	-4.434669
48	-.8421582	-4.434669
49	-.9078266	-4.434669
50	-.973773	-4.434669

Note: The above is not intended to imply 6 digit accuracy. The accuracy is about 4 digits.

Appendix 4. Some Information on Parabolic Antennas

$$\Theta \approx \frac{65\lambda}{D}$$

$$G_D \approx \frac{2.00x10^4}{\Theta^2}$$ and solving for D,

$$D \approx \frac{1}{42.1}\sqrt{G_D}$$

Θ = Beamidth
λ = Wavelength = .0517 meters at 5.8 GHz
D = Antenna Diameter
G_D = The antenna gain ratio.

D (meters, inches)	G_D	$10Log_{10}\, G_D$	note
0.25, 9.8	110.8	20.4 dB	1/4 meter
0.305, 12	164.6	22.2 dB	1 foot
0.50, 19.7	443	26.5 dB	half meter
6 feet	3259	35.1 dB	

This shows that gains in excess of 20 dBi are achievable with very small parabolic antennas. Table 5 indicates that the average gain is negative for parabolic antennas of about 10 inch diameter of more.

AVERAGE ANTENNA GAIN OF PART 15 DEVICES AS SEEN BY A LOW EARTH ORBIT SATELLITE

Jay E. Padgett
Lucent Technologies/Bell Labs
December 4, 1996

ABSTRACT

This paper addresses the issue of average antenna gain for the NII/SUPERNet devices, as seen by a satellite associated with the Mobile Satellite Service (MSS). This analysis is intended to answer the question of whether the NII/SUPERNet power output limit should be specified as a maximum EIRP, or as a maximum RF power level into the antenna terminals. MSS interests have raised the concern that the latter would allow high EIRP, and that if high-gain antennas are systematically directed horizontally (as would be expected), the EIRP as seen by a satellite at a low elevation angle could cause harmful interference to the MSS forward link.

The results show that such a concern is unjustified. In fact, the opposite is true. The average gain seen by the satellite is actually lower for high gain antennas. Three NII/SUPERNet antenna types are compared: (1) the antenna pattern introduced by AirTouch in its Reply Comments, with a range of beamwidths; (2) a parabolic dish with various diameters; and (3) a half-wave dipole. It was found that for types (1) and (2), the higher the maximum gain, the lower the average gain. For antennas with any significant directivity, the average gain was found to be less than 0 dBi. Of all the antennas studied, the half-wave dipole exhibited the highest average gain (about 0.8 dBi). This is because the high-gain antennas direct most of their energy below the 10° minimum elevation angle of the satellite.

It is concluded that it is transmit power, not EIRP, that should be regulated in the Commission's Rules, since it is the maximum transmit power that determines the potential for interference to the MSS forward link. With a transmit power limit, the interference to MSS is likely to be *less* than with an EIRP limit, because a transmit power requirement would provide some incentive for designers to use directive antennas, for which the average power to the satellite would be less than for a non-directive antenna with the same input power.

INTRODUCTION

The band 5150-5250 MHz is allocated to feeder uplinks in the Mobile Satellite Service (MSS). The FCC has proposed to also allow unlicensed operation of low-power "NII/SUPERNet" devices in that band under Part 15. A concern has been raised about the potential for interference from the Part 15 devices to the satellites. To accurately assess the restrictions that need to be applied to the Part 15 devices to avoid such interference, the impact of the Part 15 antenna pattern on the power received by the satellite needs to be understood. The basic question is whether power limit for Part 15 devices should be expressed in terms of total RF power output (i.e., into the antenna terminals), or effective isotropic radiated power (EIRP), which limits the product of the power output and the maximum antenna gain.

The purpose of this paper is to address the question of whether the total power from the Part 15 devices as seen by the satellite depends on the power output or the EIRP.

GEOMETRY AND NOTATION

Figure 1 shows the geometry of the situation.

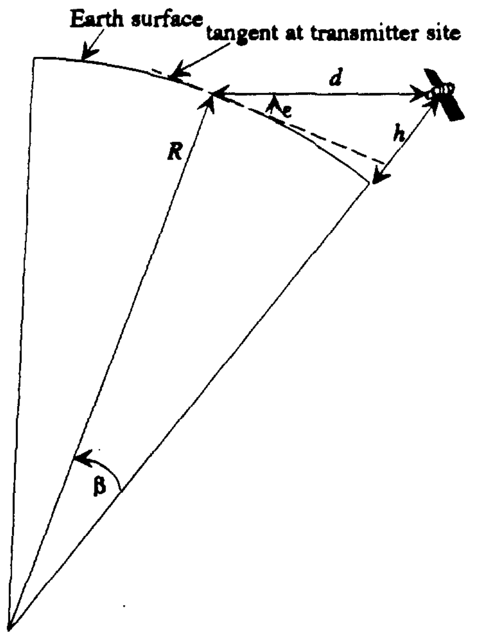


Figure 1

R is the effective radius of the Earth, allowing for atmospheric diffraction. If r is the actual radius (3963 mi), then $R = Kr$, where $K = 4/3$ represents normal diffraction, and will be used in the calculations in this paper. The satellite is h miles above the surface; in this case, $h = 879$ mi. The elevation angle of the satellite as seen by a device on the surface is ε, and the line-of-sight distance from the device to the satellite is d. The angle subtended by the surface arc between the device and the point on the surface directly below the satellite is β.

Figure 2 shows the coordinates used in the analysis. The antenna boresight (direction of maximum gain) is oriented parallel to the surface. The orientation of the satellite relative to the boresight is described by the elevation angle ε and the azimuth angle α. For an antenna with a pattern that is a figure-of-revolution (such as the parabolic dish shown in Fig. 2), the gain is a function of ϕ, which is the polar angle between the boresight and the satellite, with $\cos\phi = \cos\alpha \cdot \cos\varepsilon$. For completeness, the angle θ represents revolution about the boresight.

ANTENNA GAIN PATTERN

Consider the *radiation intensity* $U(\alpha, \varepsilon)$ in watts/steradian emanating from an antenna at the center of a sphere. Clearly, integrating $U(\alpha, \varepsilon)$ over the surface of the sphere (4π steradians) should yield the total radiated power P (the power into the antenna terminals, minus losses):

$$\int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} U(\alpha, \varepsilon) \cos\varepsilon \, d\varepsilon \, d\alpha = P \,. \tag{1}$$

Therefore, for an isotropic antenna, $U_i(\alpha,\varepsilon) = U_i = P/4\pi$.

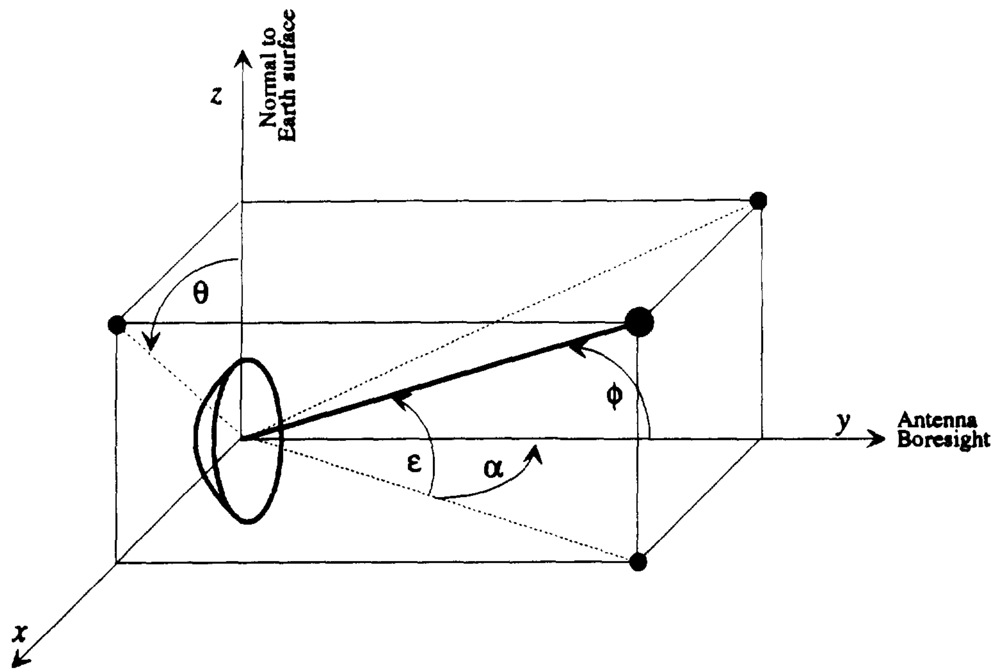


Figure 2: *Spherical coordinates*

The antenna gain pattern $G(\alpha,\varepsilon)$ gives the radiation intensity in the (α,ε) direction relative to that which would result from an isotropic antenna with the same total radiated power. Thus, $G(\alpha,\varepsilon) = U(\alpha,\varepsilon)/U_i = 4\pi U(\alpha,\varepsilon)/P$, and

$$\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} G(\alpha,\varepsilon)\cos\varepsilon\, d\varepsilon\, d\alpha = 4\pi . \tag{2}$$

The *effective power* radiated in the (α,ε) direction, relative to that from an isotropic antenna, is $P\cdot G(\alpha,\varepsilon)$. The maximum value of $G(\alpha,\varepsilon)$ is the "gain" or "directivity" of the antenna. The EIRP is normally taken to mean $P\cdot G(\alpha,\varepsilon)_{\max}$.

Of interest here is the average antenna gain, as seen by the satellite, of a terrestial transmitter with some anisotropic antenna pattern. The azimuth angle of the transmitter is assumed to be uniformly-distributed between $-\pi$ and π radians. The elevation angle ε can vary between some minimum ε_0 and $\pi/2$. For the case of interest here, $\varepsilon_0 = 10°$ ($\pi/18$ radians). However, the distribution is not uniform, because some elevation angles are more likely than others. Let $f_\varepsilon(x)$, $\varepsilon_0 \le x \le \pi/2$, be the probability density function (pdf) of the elevation angle, with

$$\int_{\varepsilon_0}^{\pi/2} f_\varepsilon(x)dx = 1 \tag{3}$$

The average gain as seen by the satellite is:

$$G_{av} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{\varepsilon_0}^{\pi/2} G(\alpha,\varepsilon)f_\varepsilon(\varepsilon)d\varepsilon d\alpha\ . \tag{4}$$

Note that for an isotropic antenna, $G_{av} = 1$.

THE AIRTOUCH ANALYSIS

In its Reply Comments in ET Docket 96-102, AirTouch attempted to compute the average gain for an antenna with a gain pattern:

$$G_{AT}(\alpha,\varepsilon) = \frac{27000}{B_\alpha B_\varepsilon}10^{-\frac{1}{2}\left[\left(\frac{\varepsilon}{B_\varepsilon}\right)^2+\left(\frac{\alpha}{B_\alpha}\right)^2\right]}+1 \tag{5}$$

It is clear from inspection that this is not a valid antenna pattern, because it violates (2). As shown in Appendix A,

$$\begin{aligned}\frac{1}{4\pi}\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} G_{AT}(\alpha,\varepsilon)\cos\varepsilon\, d\alpha\, d\varepsilon &\cong 1+\frac{27000(\pi/180)^2}{2\ln 10}e^{-B_\varepsilon^2/2\ln 10}\\ &= 1+1.786e^{-B_\varepsilon^2/2\ln 10}\end{aligned} \tag{6}$$

This has been verified with numerical integration. In dB, (6) agrees exactly with the numerical results to three decimal places for $B_\varepsilon \le 40°$, and the error is less than 0.1 dB up to 75°. B_α has little effect as long as it is less than 90°. As can be seen from (6), the excess gain ranges from about 4.4 dB for small B_ε down to about 3.8 dB for $B_\varepsilon = 60°$. The AirTouch antenna gain formula must be divided by (6) so that (2) is satisfied.

AirTouch uses (4) to evaluate the average gain, with $f_\varepsilon(\varepsilon) = \cos\varepsilon/(1-\sin\varepsilon)$, which is a valid pdf (but does not represent a uniform distribution of transmitters over the Earth's surface, as shown below). The AirTouch average gain formula is:

$$G_{ATav} = \frac{1}{2\pi(1-\sin\varepsilon)}\int_{-\pi}^{\pi}\int_{\varepsilon_0}^{\pi/2} G(\alpha,\varepsilon)\cos\varepsilon\, d\varepsilon d\alpha \tag{7}$$